Problem Set 2

Please submit the answers to me in one pdf file by **Dec 24, 2025** to my email luyunfeng@nju.edu.cn.

1. *Two-person beauty contest game*

In our class we discussed the beauty contest game with N person (N>2). Let's consider the case when N=2. The other parameters are the same: each player chooses the number from [0, 100], and the winning number will be the closest one to 2/3*mean of all the chosen numbers.

(a) What is the Nash equilibrium of this game? What is the weakly dominant strategy for each player?
(b) What is the L0 player's strategy? What is the L1 player's optimal strategy?
(c) Would you expect that the chosen numbers will be lower in this 2-person beauty contest game compared those in the N-person game (N>2)? Why?

2. *Market entry game and level-k model*

Suppose three identical, risk-neutral firms must decide simultaneously and irreversibly whether to enter a new market which can accommodate only two of them. If all three firms enter, all get payoff 0; otherwise, entrants get 9 and firms that stay out get 8.

(a) Identify the unique mixed-strategy equilibrium and describe the resulting probability distribution of the total ex post number of entrants. (You are not asked to show this, but the game also has three pure-strategy equilibria, in each of which exactly two firms enter; but these equilibria are arguably unattainable in a one-shot game in the absence of prior agreement or precedent. The mixed-strategy equilibrium is symmetric, hence attainable.)

Now suppose that each firm follows a behavioral rule that is an independent and identically distributed draw from a distribution that assigns equal probabilities to two types: either L1 (best response assuming the other firms are each equally likely to enter or stay out, and probabilistically independent), or L2 (best response to L1).

(b) Describe the decisions of types L1 and L2 and the resulting actual (as opposed to what L1 or L2 expect) probability distribution of the total ex post number of entrants when each firm's type is drawn as explained above. Show that the expected number of entrants is closer to the ex post optimal number (2) than in your equilibrium from part (a), and that that the probability of exactly 2 entrants is higher than in (a). (In experiments subjects' initial responses come systematically closer to ex post optimality than the symmetric mixed-strategy equilibrium predicts, a result Kahneman has described as "magic." This analysis shows that bounded strategic rationality works like fairy dust.)

(The following (c) and (d) are **bonus questions, not required**) Now suppose that each firm follows a rule that is an independent and identically distributed draw from a distribution that assigns probability ½ to type L1, ¼ to L2, and ¼ to a type called Sophisticated, which plays an equilibrium in the game in which the prior probabilities of L1, L2, and Sophisticated players are common knowledge.

(c) Plugging in the behaviors of L1 and L2 players (which do not depend on the prior type probabilities), characterize equilibrium in the game played by Sophisticated players.

(d) How does your answer to (c) change, if at all, if the prior probability of Sophisticated players is $\varepsilon \approx 0$, and the prior probability of L2 players is ½ -ε (with the prior probability of L1 players held constant at ½)?